MR2642159 (2011h:14049) 14J32 (14C25 14C30 14F43)
Del Angel, Pedro Luis [del Angel Rodríguez, Pedro Luis] (MEX-CIM);
Müller-Stach, Stefan [Müller-Stach, Stefan J.] (D-MNZ-IM);
Van Straten, Duco (D-MNZ-IM); **Zuo, Kang [Zuo, Kang[1]]** (D-MNZ-IM)
Hodge classes associated to 1-parameter families of Calabi-Yau 3-folds. (English summary)
Acta Math. Vietnam. **35** (2010), *no. 1,* 7–22.

This paper presents a detailed description of the Hodge numbers of the L^2-cohomology group $H^1_{(2)}(S,\mathbb{V})$, where $f\colon X\to S$ is a family of Calabi-Yau threefolds over a smooth complex curve S and $\mathbb{V}=R^3f_*(\mathbb{C}_X)$. For $\overline{f}\colon\overline{X}\to\overline{S}$ a semi-stable completion of f, let $D=\overline{S}\smallsetminus S$ denote the image of the singular fibers of $\overline{f}$, and set $\Delta=\overline{f}^{-1}(D)$. To obtain their results the authors analyze the structure of the logarithmic Higgs bundle (E,θ) on $\overline{S}$, where E is a direct sum of Hodge bundles $E^{p,q}=R^q\overline{f}_*(\Omega^p_{\overline{X}/\overline{S}}(\log\Delta))$, with $p+q=3$ and $\dim E^{p,q}=1$. The Higgs field $\theta\colon E\to E\otimes\Omega^1_{\overline{S}}(\log D)$ is obtained from the cup product with the Kodaira-Spencer class. Note that by Griffiths transversality θ maps $E^{p,q}$ into $E^{p-1,q+1}\otimes\Omega^1_{\overline{S}}(\log D)$. The authors show that principal examples arise from nontrivial pencils of Calabi-Yau threefolds, a number of which are expressed in the paper's final section.

By the work of J. Jost, Y. H. Yang and K. Zuo [J. Reine Angew. Math. **609** (2007), 137–159; MR2350782 (2009e:32021)], which extends S. M. Zucker's results in [Ann. of Math. (2) **109** (1979), no. 3, 415–476; MR0534758 (81a:14002)] on metrized local systems to Higgs bundles, one deduces that the L^2-Higgs complex $(\Omega^\bullet_{(2)}(E),\theta)$ associated to (E,θ) is a resolution of $j_*\mathbb{V}$. Here $j\colon S\hookrightarrow\overline{S}$ denotes the inclusion. In particular there are isomorphisms $H^k_{(2)}(S,\mathbb{V})\cong H^k(\overline{S},j_*\mathbb{V})\cong\mathbb{H}^k(\Omega^\bullet_{(2)}(E),\theta)$.

Using the residue of the Higgs field at each boundary point $p\in D$, which is expressed as the nilpotent endomorphism of E_p obtained from the monodromy logarithms, the authors classify the singular values of $\overline{f}$ and give an explicit description of the Hodge spaces $H^{p,q}$ of $H^1_{(2)}(S,\mathbb{V})$. In the penultimate section of the paper the authors discuss algebraic cycles $Z\in\mathrm{CH}^2(\overline{X})$, which are homologous to zero on the fibers, in terms of extensions of Higgs bundles.

{Reviewer's remark: There is an obvious misprint in the description of $H^{1,3}$ in Theorem 2.1 on page 21.}

Reviewed by *Michael J. Paluch*

References

1. G. Almkvist, C. van Enckevort, D. van Straten and W. Zudilm, Tables of Calabi-Yau equations, arXiv:math/0507430.
2. P. Candelas, X. de la Ossa, P. S. Green and L. Parkes, An exactly soluble superconformal theory from a mirror pair of Calabi-Yau manifolds, *Physics Letters B* **258** (1991, 118–126. MR1101784 (93b:32028)

3. P. L. del Angel, S. Müller-Stack, Differential equations associated to families of algebraic cycles, *Annales de l'Institut Fourier,* Tome 58 (2008), Fasc. 6, 2075–2085. MR2473629 (2010a:34202)
4. C. Doran and J. Morgan, Mirror symmetry and integral variations of Hodge structure underlying one parameter families of Calabi-Yau threefolds, In: *Mirror Symmetry V,* Proceedings of BIRS workshop on Calabi-Yau Varieties and Mirror Symmetry (2003), *AMS/IP Stud. Adv. Math.* **38,** Amer. Math. Soc., Providence (2006), 517–537. MR2282973 (2008e:14010)
5. V. Batyrev, I. Ciocan-Fontanine, B. Kim and D. van Straten, Conifold transitions and mirror symmetry for Calabi-Yau complete intersections in Grassmannians, *Nuclear Phys. B* **514** (1998), 640–666. MR1619529 (99m:14074)
6. Ph. Griffiths, Infinitesimal variations of Hodge structure III. Determinantal varieties and infinitesimal invariants of normal functions, *Comp. Math.* **50** (2–3) (1983), 267–324. MR0720290 (86e:32026c)
7. J. Jost, Y.-H. Yang and K. Zuo, Cohomologies of unipotent harmonic bundles over noncompact curves, *J. reine angew. Math.* **609** (2007), 137–159. MR2350782 (2009e:32021)
8. C. Meyer, The mirror quintic as a quintic, arXiv:math/0503329.
9. P. Stiller, *Automorphic Forms and the Picard Number of an Elliptic Surface,* Aspects of Mathematics, Vol. E5, Vieweg Verlag, 1984. MR0765222 (87c:11049)
10. P. Stiller, Special values of Dirichlet series, monodromy, and the periods of automorphic forms, *Memoirs ofthe AMS, Nr.* **299** (1984). MR0743544 (86f:11043)
11. J. Walcher, *Opening Mirror Symmetry on the Quintic,* arXiv:hep-th/0605162v2. MR2350434 (2008m:14111)
12. J. Walcher, *Calculations for Mirror Symmetry with D-branes,* arXiv:0904.4905. MR2580683
13. S. Zucker, Hodge theory with degenerating coefficients, *Annals of Mathematics* **109** (2) (1979), 415–476. MR0534758 (81a:14002)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.